Mathematical Analysis of Suction and Surfactant in Microdroplets.

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Abstract
Microdroplets have emerged as a versatile platform for chemical and biological reactions, with applications in fields such as drug delivery, synthesis and cosmetic applications. However, the dynamics of suction and surfactants in microdroplets remain poorly understood, hindering the optimization of the systems. This study presents a mathematical analysis of suction and surfactant dynamics in microdroplets, with a focus on the role of surfactants as determinants of the rate of reaction. The model captures the interplay between surfactant transport, interfacial tension, and fluid flow, revealing the complex mechanisms governing microdroplet behavior. As the suction parameter (V_0) *increases the velocity also increases but the velocity reduces as the surface tension increases. The presence of surfactant at interface reduces the surface force. The results of this study have important applications for the design and optimization of microdroplet-based systems.*

Key words*: Surface tension, pressure gradient, surfactant, micro droplets and Suction Parameter*.

Introduction

Droplets of water is very important in both living and nonliving thing since life totally depends on it either for direct consumption or for cleaning. Human being for example depends on water and fluids for daily activities and the use of droplets or its combinations will be properly monitor because it can go a long way to affect major part of the society. The adoption of an infinitesimal fluid element was considered to be acting upon by two types of forces, namely, body force and surface force which is the same as Tension at the interface. Surface force adjustment could lead to the response amplitude for the case of moving objects which is not affected by increase in mass of the flow load Usman et al., (2020). Accurate modeling of such flows is challenging because of the discontinuity in material properties across the interface and because of interfacial boundary conditions due to surface forces. Usman *et al.*, (2019) worked on the effects of pressure gradient on rotating concentric cylinders which was later adopted as a way of improving the fluid flow in the system.

Microdroplets are tiny, isolated volumes of liquid that have become increasingly important in various fields, including chemistry, biology, and medicine. These tiny droplets can be used to confine and manipulate chemical and biological reactions, enabling applications such as screening, synthesis, and drug delivery. Droplets from human body system could be transmitted and harmful (Virus Disease outbreak) if necessary measure is not in place Bagbe et al., (2019).

However, the behavior of microdroplets is influenced by a complex interplay of physical and chemical factors, including suction, surfactants, and interfacial tension. Surfactants, in particular, play a crucial role in determining the behavior of microdroplets. These amphiphilic molecules can alter the interfacial tension between the droplet and its surroundings, influencing the rate of reaction and the stability of the droplet. Badejo *et al*., (2019) studied the magnetic field effects on reactive flow which also addresses the reaction of fluid like water reacting with other substances. The dynamics of surfactants in microdroplets remain poorly understood, and a comprehensive mathematical framework is needed to capture their behaviour. This study aims to develop a mathematical model of suction and surfactant dynamics in microdroplets, with a focus on the role of surfactants as determinants of the rate of reaction. By capturing the complex interplay between surfactant transport, interfacial tension, and fluid flow, this model will provide valuable insights into the behavior of microdroplets and inform the design and optimization of the systems.

Wu et al., (2007) studied Effects of surfactants on the formation of microdroplets in the flow focusing microfluidic device, it was observed that droplet-based in vitro compartmentalization (IVC) platform is an important tool in protein analysis. The formation of microdroplets is necessary during the modification microfluidic chip. The study critically examined the nature of surfactants on the modifications of microdroplets in the flow focusing microfluidic device which is important for enzyme formation. The droplet formation was affected, showing that the size of droplets reduces as the concentration of Surfactants increases.

Usman et al., (2015) investigated the modeling of surface tension, surfactant and suction of micro droplets. A model was developed to simulate interfacial flows and reactions of surface tension and surfactant. The continuity and momentum equations were used to formulate the governing equations for the problem. The governing equations were solved with variable separable and Eigen function expansion method. Suction parameter was varied with and without surfactants. The results obtained show that the reactions of surfactant with surface tension are in opposite direction and it shows good agreement with experimental data and theory.

Liu et al., (2009) studied the behavior of micro droplets flowing in microchannels with a series of diffuser/ nozzle structures. The deformation of the microdroplet was analyzed with a level set method. The behaviour of the water droplet and the corresponding pressure were observed across the diffuser/ nozzle structures and measured. The results show a direct relationship between the pressure drop and the flow rate.

Schramm et al., (2003), investigated surfactants and its applications. The general properties of surfactant relating to the surface force was observed but emphasis was made on the alter surface and interfacial properties for solubility and micelles. During the study of properties, surfactants in wettability modification, detergency, and the displacement of liquid phases through porous media were considered. These in turn lead to foods, and crop protection. Certain impurities, called surfactants, which have dramatic influence on surface tension because they agglomerate on the surface and form an elastic skin which resists stretching with increasing force. Best known among these are soaps and detergents from which one can blow so beautiful bubbles. A lipid membrane also surrounds every living cell and separates the internal biochemistry from the outside.

Manjula et al., (2025). Investigated safety of cosmetic products, particularly face creams applications, which is a growing concern for cosmetic users in due to the presence of harmful chemicals and unsatisfied formulations. This study presents a mathematical model to assess and enhance the safety of face creams users. The model incorporates statistical techniques and differential equations to predict potential adverse reactions based on ingredient concentrations and user demographics. The model serves as a tool for regulatory bodies to evaluate product safety

effectively which provides a scientific approach to cosmetic safety and it aims to protect consumers from hazardous formulations and promote healthier skincare choices for women.
The importance of pressure gradient and Hartmman number was addressed by (Badejo et al., 2019 & Usman et al., 2019) which will later offered potential avenues for improving clinical outcomes in skin management.
Adela et al., (2023), studied distributed parameters systems to determined models and to characterize the cosmetic creams in all stages of the manufacturing process, starting with the development of recipes, the description of raw materials, manufacturing technologies, and the determination of the physico-chemical and microbiological indicators that most strongly influence their quality. The result of the models, led to important conclusions regarding obtaining high quality in the studied creams and to the confirmation of the usefulness of applying the principles of Systems Theory to the study of cosmetic products.
Badejo et al., (2015) studied the modeling of surface tension of micro droplet with variations in pressure gradient. surface tension, pressure gradient and surfactant were considered during the analysis and the results shows that as the surfactant increases the surface force reduces for each droplets. The pressure gradient was also varied and also considered. The results obtained show that pressure gradient is smooth, and more stable and also shows a good agreement with the experimental data and theory.
This work will address the applications of surfactants and surface tension of micro droplets which can be considered in Medicine and pharmaceutical industry. This work intends to study the effect of surface tension, surfactants and examine suction in micro droplets. Several works have been done on microdroplets especially the contents or the composition of fluids like water but the reactions of surfactant has not been addressed mathematically. Therefore, this study will show the impact of surfactants on surface tension of water in terms of velocity distributions and also investigate the administrations of medication in the area of health treatment.

METHOD OF SOLUTIONS

The momentum equation presented below is an unsteady flow, dimensionalized partial differential equation of one dimension: with suction parameter (v_0), pressure gradient (G), Reynolds number (Re) and Hartmman number (Ha) in the formations of the mathematical model (system). Usman *et al*., (2020) concluded that the methods of Klein-Gordon equations is capable to converge to exact solutions with least number of iterations.

$$\frac{\partial U}{\partial t} + v_0 \frac{\partial U}{\partial y} = G + \frac{1}{Re}\frac{\partial^2 U}{\partial y^2} \tag{1}$$

The steady momentum equation was adopted

$$\frac{1}{Re}\frac{\partial^2 U}{\partial y^2} - v_0 \frac{\partial U}{\partial y} + HaU + G = 0 \tag{2}$$

Here are the Initial and Boundary conditions:

$$\frac{\partial U}{\partial y}(0) = \gamma, \quad 0 < y < 1, \quad U(1) = L \tag{3}$$

The general solution of the momentum equation of the system is as presented below:

$$U = \frac{e^{V_0 Re y}(-\lambda V_0 + \lambda)}{V_0^2 Re} + \frac{\lambda y}{V_0} + \frac{\gamma V_0^2 Re - \lambda Re V_0 - e^{V_0 Re}\lambda V_0 + \lambda e^{V_0 Re}}{V_0^2 Re}$$

RESULTS AND DISCUSSION

Figure1 shows the velocity profile of various value of suction parameter ($V_0 = 0.10 - 0.50$) with surface tension ($\gamma = 0.1$) while other parameters remain constant. As the suction parameter (V_0) increases the velocity also increases suggesting the free flow of fluid in the system.

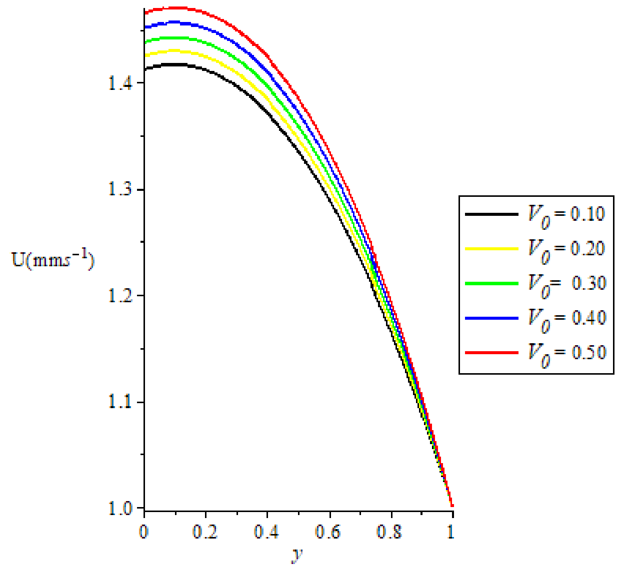

Fig. 1 Velocity profile for various values of suction parameter (V_0) = 0.10 - 0.50 with $\gamma = 0.1$, $\lambda = 1$, G = 1, Re = 1.

Figure 2 shows the velocity profile of suction parameter ($V_0 = 0.10 - 0.50$) with surface tension ($\gamma = 0.2$) while other parameters remain constant. It was observed that as the suction parameter (V_0) increases the velocity also increases. It was also observed that velocity reduces as the surface tension increases. Indicating that surface forces reduce the fluid flow in the system. Like Akanbi et al., (2024) influence and information could assist in this area.

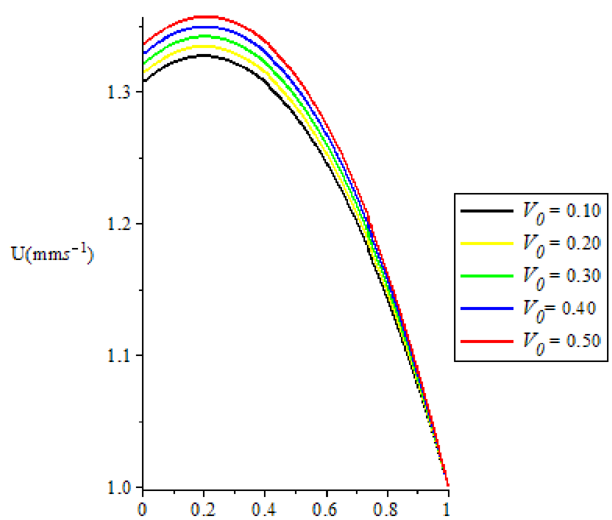

Fig. 2 Velocity profile for various values of suction parameter (V_0) = 0.10 - 0.50 with $\gamma = 0.2$, $\lambda = 1$, G = 1, Re = 1.

Figure 3 shows the velocity profile of suction parameter ($V_0 = 0.10 - 0.50$) with surface tension ($\gamma = 0.3$) while other parameters remain constant. As the suction parameter (V_0) increases the velocity also increases but velocity reduces as the surface tension increases. Indicating the resistance of fluid flow in the system.

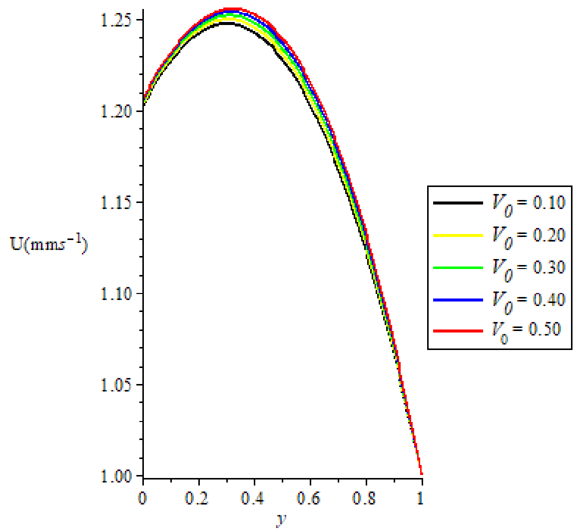

Fig. 3 Velocity profile for various values of suction parameter (V_0) = 0.10 - 0.50 with γ = 0.3, λ = 1, G = 1, Re = 1.

Figure 4 shows the velocity profile of suction parameter ($V_0 = 0.10 - 0.50$) with surface tension ($\gamma = 0.4$) while other parameters remain constant. As the suction parameter (V_0) increases the velocity also increases but the velocity reduces as the surface tension increases. Suggesting resistance of fluid flow in the system.

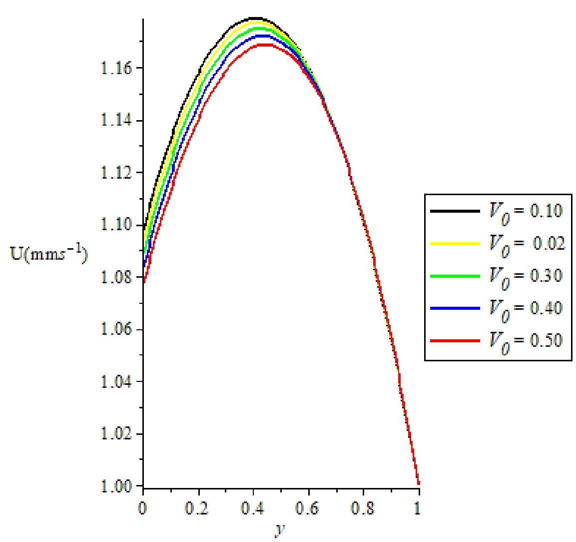

Fig. 4 Velocity profile for various values of suction parameter (V_0) = 0.10 - 0.50 with γ = 0.4, λ = 1, G = 1, Re = 1.

Figure 5 shows the velocity profile for values of surface tension ($\gamma = 0.00 - 0.40$) with surfactant value ($\lambda = 0.2$) and suction value ($V_0 = 0.1$) while other parameters remain constant. As the surface tension (γ) increases the velocity also reduces. It was observed that there was free flow of fluid in the system, suggesting that there is more reactions of fluid as a result of surfactant in the system.

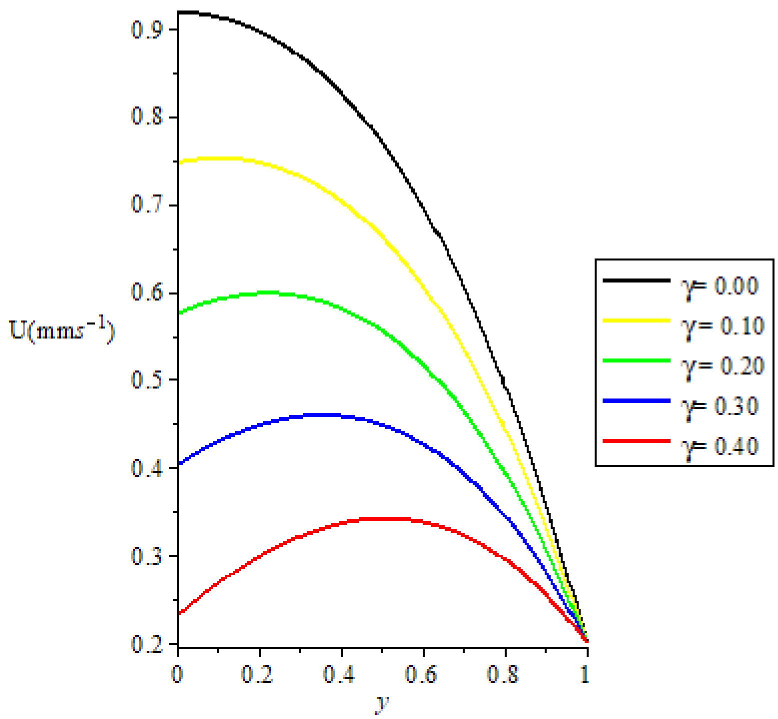

Fig. 5 Velocity profile for various values of surface tension (γ) = 0.00 - 0.40 with V_0 = 0.1, λ = 0.2, G = 1, Re = 1.

Figure 6 shows the velocity profile for values of surface tension ($\gamma = 0.00 - 0.40$) with surfactant value ($\lambda = 0.4$) and suction value ($V_0 = 0.1$) while other parameters remain constant. As the surface tension (γ) increases the velocity also reduces. As Suggesting that the velocity of the system reduces as a result of increase in surface force in the system. It was observed that the rate of reaction increases as surfactant (λ) value increases. Free flow of fluid in the system reduces the body temperature Badejo et al., (2024)

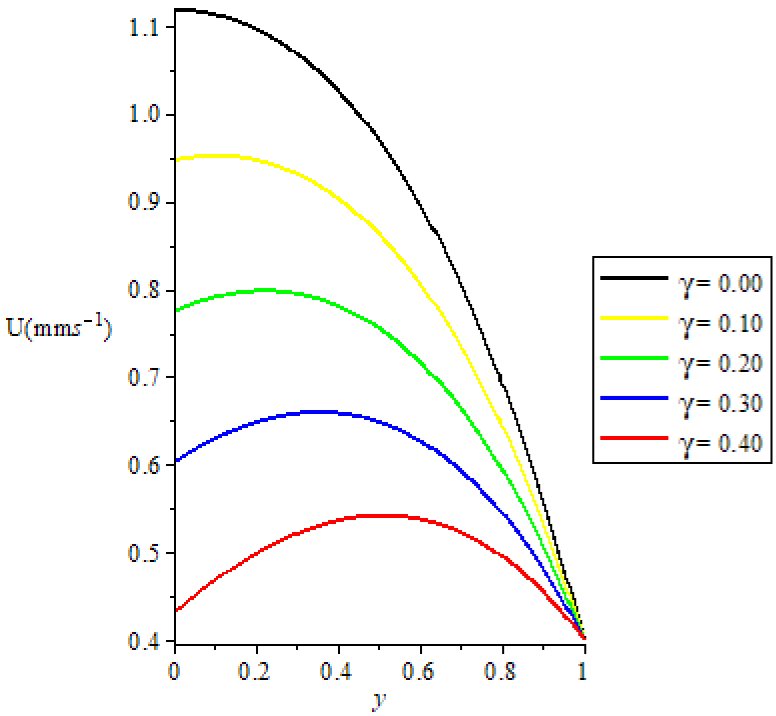

Fig. 6 Velocity profile for various values of surface tension $(\gamma) = 0.00 - 0.40$ with $V_0 = 0.1$, $\lambda = 0.4$, G = 1, Re = 1.

Figure 7 shows the velocity profile for various value of surface tension ($\gamma = 0.00 - 0.40$) with surfactant value ($\lambda = 0.6$) and suction value ($V_0 = 0.1$) while other parameters remain constant. As the surface tension (γ) increases the velocity also reduces. Suggesting that the velocity of the system reduces as a result of increase in surface force of the system. It was also observed that the rate of reaction increases as surfactant (λ) value increases.

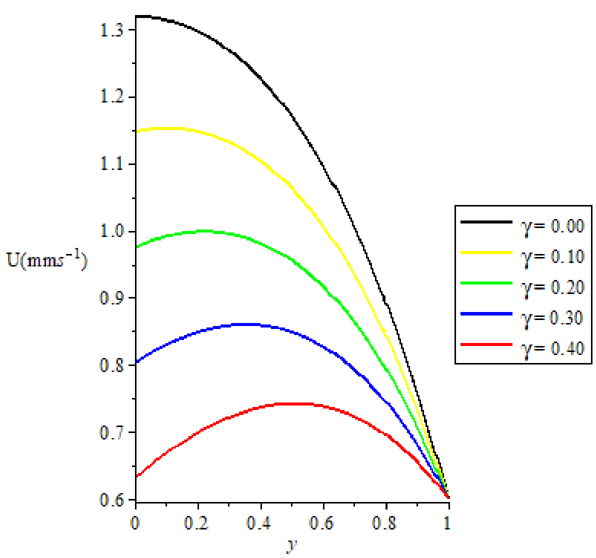

Fig. 7 Velocity profile for various values of surface tension $(\gamma) = 0.00 - 0.40$ with $V_0 = 0.1$, $\lambda = 0.6$, G = 1, Re = 1.

Figure 8 shows the velocity profile for values of surface tension ($\gamma = 0.00 - 0.40$) with surfactant value ($\lambda = 0.8$) and suction value ($V_0 = 0.1$) while other parameters remain constant. As the surface tension (γ) increases the velocity also reduces. Suggesting that the velocity of the system reduces as a result of increase in surface force of the system. It was observed that the rate of reaction increases as surfactant (λ) value increases.

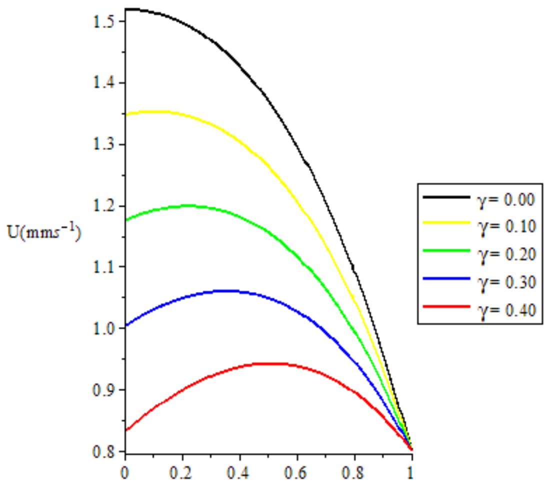

Fig. 8 Velocity profile for various values of surface tension (γ) = 0.00 - 0.40 with V_0 = 0.1, λ = 0.8, G = 1, Re = 1.

CONCLUSION

The velocity of the system reduces as a result of increase in surface force of the system. It was observed that the rate of reaction increases as surfactant (λ) value increases. The presence of surfactant at interface reduces the surface tension.

Reducing interfacial tension improves the stability of a droplet: which is in line with improving the contact between the drug or any skin application and skin surface leading to permeability of skin applications administering.

It was observed that there was free flow of fluid in the system, suggesting that there is more reactions of fluid as a result of surfactant in the system.

As the suction parameter (V_0) increases the velocity also increases but the velocity reduces as the surface tension increases. Suggesting resistance of fluid free flow in the system.

Generally, administering any skin applications should be well observed with time (regular cleaning or remover of skin applications should be done regularly because of the permeability of skin) due to the surface area of skin types. Some products have low surface tensions and spread rapidly and easily on the surface of skin, while others are difficult to apply to the surface.

Pressure gradient (water drops of different radii determine the stability of the skin after its application) We also observed that low value of surfactant has no effect on higher value of surface tension at interface of homogenous fluids.

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